

**GDR 134**

**GROUPEMENT DE RECHERCHE  
TRAITEMENT DU SIGNAL ET IMAGES**



# CONTENTS

## Digital Signal Processing Tools

<b>addnc :</b>	Addition of two non causal signals
<b>costh :</b>	Entropic cost function
<b>dec2nc :</b>	Decimation by a factor of 2 of a non causal signal
<b>filtnc :</b>	Non causal filtering of a non causal signal
<b>filtnc_p :</b>	Non causal filtering of a non causal periodized signal
<b>int2nc :</b>	Interpolation with zeros of a non causal signal
<b>lowpow2 :</b>	Biggest power of 2 value less than or equal to the length of a signal
<b>nfft :</b>	Normalized DFT
<b>nifft :</b>	Normalized inverse DFT
<b>polypow :</b>	Power of a polynomial

## Wavelet Decompositions

<b>dwt :</b>	Discrete wavelet decomposition
<b>idwt :</b>	Inverse discrete wavelet decomposition
<b>pdwt :</b>	Periodic discrete wavelet decomposition
<b>pdwt_it :</b>	Periodic discrete wavelet decomposition computed over one iteration
<b>ipdwt :</b>	Inverse periodic discrete wavelet decomposition
<b>ipdwt_it :</b>	Inverse periodic discrete wavelet decomposition computed over one iteration
<b>cdwt :</b>	Redundant discrete wavelet decomposition
<b>pcdwt :</b>	Redundant periodic discrete wavelet decomposition
<b>bdwtd :</b>	Discrete wavelet decomposition on the interval
<b>bwaveld :</b>	Generation of left boundary wavelet coefficients
<b>bwaverd :</b>	Generation of right boundary wavelet coefficients
<b>ibdwt :</b>	Inverse discrete wavelet decomposition on the interval
<b>ibwaveld :</b>	Generation of the left boundary matrices used in the inverse wavelet decomposition on the interval
<b>ibwaverd :</b>	Generation of the right boundary matrices used in the inverse wavelet decomposition on the interval

## Wavelet Packet Decompositions

<b>dwpt :</b>	Discrete wavelet packet decomposition
<b>idwpt :</b>	Inverse discrete wavelet packet decomposition
<b>pdwpt :</b>	Periodic discrete wavelet packet decomposition
<b>ipdwpt :</b>	Inverse periodic discrete wavelet packet decomposition

## Design Tools

<b>exmeyer :</b>	Example function to construct Meyer's wavelet
<b>freqcqf :</b>	Complex conjugate frequency response of a filter
<b>frmeyer :</b>	Frequency response of Meyer's QMF filter
<b>frsplis :</b>	Frequency responses of lowpass B-spline symmetric wavelet filters
<b>gene_pw :</b>	Generation of filters H and G for periodic discrete wavelet decomposition.
<b>irdaub :</b>	Impulse response of Daubechies wavelet filters
<b>irsplis :</b>	Impulse response of lowpass B-spline symmetric wavelet filters
<b>pdwav :</b>	Generation of discrete periodic wavelets and scaling functions
<b>pdwavp :</b>	Generation of discrete periodic wavelet packets
<b>qmf :</b>	Conjugate quadrature mirror filter

## Wavelet Toolbox Demonstrations

- dwtdemo** : Example script for wavelet decomposition
- dwptdemo** : Example script for wavelet packet decomposition
- bdwtdemo** : Example script for boundary wavelet decomposition

# **Digital Signal Processing Tools**

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<b>filtnc_p :</b>	Non causal filtering of a non causal periodized signal
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## **addnc**

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### **Purpose**

Addition of two non causal signals

### **Synopsis**

$[z,tz] = \text{addnc}(x,tx,y,ty)$

### **Description**

$[z,tz] = \text{addnc}(x,tx,y,ty)$  adds two non causal signals  $x$  and  $y$  starting at time  $tx$  and  $ty$ , respectively.

### **Authors**

J-C. Pesquet and H. Krim

## **costh**

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### **Purpose**

Entropic cost function

### **Synopsis**

$c = \text{costh}(x)$

### **Description**

$c = \text{costh}(x)$  computes the entropic cost of signal  $x$ .

### **Authors**

J-C. Pesquet and H. Krim

## **dec2nc**

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### **Purpose**

Decimation by a factor of 2 of a non causal signal

### **Synopsis**

$[y,ty] = \text{dec2nc}(x,tx)$

### **Description**

$[y,ty] = \text{dec2nc}(x,tx)$  decimates the incoming series  $x$  starting at time  $tx$ .  $ty$  is the time delay of  $y$ .

### **See also**

INT2NC in the GDR Wavelet Toolbox.

### **Authors**

J-C. Pesquet and H. Krim

## **filtnc**

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### **Purpose**

Non causal filtering of a non causal signal

### **Synopsis**

$[y,ty] = \text{filtnc}(b,tb,x,tx)$

### **Description**

$[y,ty] = \text{filtnc}(b,tb,x,tx)$  performs a non causal filtering of the non causal signal  $x$ , starting at time  $tx$ , using the filter  $b$  with time delay  $tb$ .  $y$  is the filtered signal which starts at time  $ty$ .

### **See also**

FILTNC\_P in the GDR Wavelet Toolbox.

### **Authors**

J-C. Pesquet and H. Krim

## **filtnc\_p**

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### **Purpose**

Non causal filtering of a non causal periodized signal

### **Synopsis**

$y = \text{filtnc\_p}(b, tb, x, tx)$

### **Description**

$y = \text{filtnc\_p}(b, tb, x, tx)$  performs non causal filtering of the non causal periodized signal  $x$ , starting at time  $tx$ , using the filter  $b$  whose time delay is  $tb$ .

### **See also**

FILTNC in the GDR Wavelet Toolbox.

### **Authors**

J-C. Pesquet and H. Krim

## **int2nc**

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### **Purpose**

Interpolation with zeros of a non causal signal

### **Synopsis**

$[y,ty] = \text{int2nc}(x,tx)$

### **Description**

$[y,ty] = \text{int2nc}(x,tx)$  interpolates with zeros the non causal signal  $x$ .  $y$  is twice length of  $(x -1)$  long.  $tx$  is the delay of the original signal and  $ty$  the delay of the interpolated signal.

### **See also**

DEC2NC in the GDR Wavelet Toolbox.

### **Authors**

J-C. Pesquet and H. Krim

## **lowpow2**

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### **Purpose**

Compute the biggest power of 2 value less than or equal to the length of a signal

### **Synopsis**

$y = \text{lowpow2}(x)$

### **Description**

$y = \text{lowpow2}(x)$  returns the biggest power of 2 value less than or equal to  $x$ .

### **Authors**

J-C. Pesquet and H. Krim

## **nfft**

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### **Purpose**

Compute the normalized Discrete Fourier Transform

### **Synopsis**

$y = \text{nfft}(x,n)$

### **Description**

$y = \text{nfft}(x,n)$  computes the normalized DFT where  $x$  is the input data, and  $n$  is the number of samples. By default  $n$  is equal to  $\text{length}(x)$ .

### **See also**

NIFFT in the GDR Wavelet Toolbox.

### **Authors**

J-C. Pesquet and H. Krim

## **nifft**

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### **Purpose**

Compute the inverse normalized Discrete Fourier Transform

### **Synopsis**

$y = \text{nifft}(x,n)$

### **Description**

$y = \text{nifft}(x,n)$  computes the inverse normalized DFT where  $x$  is the input data, and  $n$  is the number of samples. By default  $n$  is equal to  $\text{length}(x)$ .

### **See also**

NFFT in the GDR Wavelet Toolbox.

### **Authors**

J-C. Pesquet and H. Krim

## **polypow**

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### **Purpose**

Compute the power of a polynomial

### **Synopsis**

$c = \text{polypow}(b,n)$

### **Description**

$c = \text{polypow}(b,n)$  computes the power  $n$  of polynomial  $b$ .

### **Authors**

J-C. Pesquet and H. Krim

## **Wavelet Decompositions**

## Contents

<b>dwt :</b>	Discrete wavelet decomposition
<b>idwt :</b>	Inverse discrete wavelet decomposition
<b>pdwt :</b>	Periodic discrete wavelet decomposition
<b>pdwt_it :</b>	Periodic discrete wavelet decomposition computed over one iteration
<b>ipdwt :</b>	Inverse periodic discrete wavelet decomposition
<b>ipdwt_it :</b>	Inverse periodic discrete wavelet decomposition computed over one iteration
<b>cdwt :</b>	Redundant discrete wavelet decomposition
<b>pcdwt :</b>	Redundant periodic discrete wavelet decomposition
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<b>bwaveld :</b>	Generation of left boundary wavelet coefficients
<b>bwaverd :</b>	Generation of right boundary wavelet coefficients
<b>ibdwt :</b>	Inverse discrete wavelet decomposition on the interval
<b>ibwaveld :</b>	Generation of the left boundary matrices used in the inverse wavelet decomposition on the interval
<b>ibwaverd :</b>	Generation of the right boundary matrices used in the inverse wavelet decomposition on the interval

## **bdwtd**

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### **Purpose**

Discrete wavelet decomposition using boundary wavelets

### **Synopsis**

$y = \text{bdwtd}(h,g,hl,gl,AI,hr,gr,Ar,x,rm)$

### **Description**

$y = \text{bdwtd}(h,g,hl,gl,AI,hr,gr,Ar,x,rm)$  computes the discrete wavelet decomposition of the signal  $x$  using boundary wavelets.  $h$  and  $g$  are the lowpass and highpass QMFs whose impulse responses are taken between  $-n+1$  and  $n$  where  $n = \text{length}(h)/2$ .  $hl$  is the matrix of the lowpass coefficients for the left boundary,  $gl$  is the matrix of the highpass coefficients for the left boundary,  $AI$  is the preconditioning matrix for the left boundary,  $hr$  is the matrix of the lowpass coefficients for the right boundary,  $gr$  is the matrix of the highpass coefficients for the right boundary and  $Ar$  is the preconditioning matrix for the right boundary.  $x$  is the data vector and  $rm$  is the maximum resolution level.  $y$  is the vector of approximation coefficients at the lowest level and wavelet coefficients stored according to increasing resolution order.

### **See also**

IBDWTD, BWAVELD, BWAVERD in the GDR Wavelet Toolbox.

### **Authors**

J-C. Pesquet and H. Krim

### **References**

[1] A. Cohen and I. Daubechies, "Wavelets on the interval and Fast Wavelet Transform", Applied and Computational Harmonic Analysis 1, 54-81, 1993.

## **bwaveld**

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### **Purpose**

Generation of left boundary wavelet coefficients

### **Synopsis**

$[H,G,A] = \text{bwaveld}(h)$

### **Description**

$[H,G,A] = \text{bwaveld}(h)$  generates the coefficients for the left boundary wavelets of the lowpass filter  $h$  whose impulse response is taken between  $-N_2+1$  and  $N_2$  where  $N_2 = \text{length}(h)/2$ .  $H$  is the matrix of the lowpass coefficients for the left boundary,  $G$  is the matrix of the highpass coefficients for left boundary and  $A$  is the preconditioning matrix for left boundary data.

### **See also**

BDWTD, BWAVERD in the GDR Wavelet Toolbox.

### **Authors**

J-C. Pesquet and H. Krim

### **References**

[1] A. Cohen and I. Daubechies, "Wavelets on the interval and Fast Wavelet Transform", Applied and Computational Harmonic Analysis 1, 54-81, 1993.

## **bwaverd**

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### **Purpose**

Generation of right boundary wavelet coefficients

### **Synopsis**

$[H,G,A] = \text{bwaverd}(h)$

### **Description**

$[H,G,A] = \text{bwaverd}(h)$  generates the coefficients for the right boundary wavelets of the lowpass filter  $h$  whose impulse response is taken between  $-N_2+1$  and  $N_2$  where  $N_2 = \text{length}(h)/2$ .  $H$  is the matrix of the lowpass coefficients for the right boundary data,  $G$  is the matrix of the highpass coefficients for the right boundary data and  $A$  is the preconditioning matrix for the right boundary data.

### **See also**

BDWTD, BWAVELD in the GDR Wavelet Toolbox.

### **Authors**

J-C. Pesquet - H. Krim

### **References**

[1] A. Cohen and I. Daubechies, "Wavelets on the interval and Fast Wavelet Transform", Applied and Computational Harmonic Analysis 1, 54-81, 1993.

## cdwt

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### Purpose

Redundant discrete wavelet decomposition

### Synopsis

$[y,ty] = \text{cdwt}(H,tH,G,tG,x)$

$[y,ty] = \text{cdwt}(H,tH,G,tG,x,rm)$

### Description

$[y,ty] = \text{cdwt}(H,tH,G,tG,x)$  computes the redundant discrete wavelet decomposition vector  $y$  of the signal  $x$ .  $y$  is a vector where the details of  $x$  are stored from coarse to fine resolution.  $H$  and  $G$  are the impulse responses of the analysis lowpass and highpass filters starting at time  $tH$  and  $tG$ , respectively, and  $ty$  is a matrix containing the following information :

- 1st row: delays of  $y$  signals
- 2nd row: length of  $y$  signals

$[y,ty] = \text{cdwt}(H,tH,G,tG,x,rm)$  computes the redundant discrete wavelet decomposition up to the resolution  $rm$ .

### Algorithm

The discrete wavelet decomposition function is computed using the pyramid algorithm which is described by Mallat in [1]. This algorithm consists in decomposing the signal spectrum into two equal (orthogonal case) or unequal (biorthogonal case) frequency bands using a lowpass ( $H$ ) and a highpass ( $G$ ) quadrature mirror filters (QMFs). They are called the approximations vector and the details vector, respectively. Then, the same procedure is applied again to the approximations vector as many times -resolution levels- as it is possible. But, the results of  $H$  and  $G$  are not decimated by 2, and therefore the number of generated wavelet coefficients increases exponentially with the number of resolution levels.

### See also

PCDWT in the GDR Wavelet Toolbox.

### Authors

J-C. Pesquet and H. Krim

### References

- [1] S. G. Mallat, "A Theory for Multiresolution Signal Decomposition: The Wavelet Representation," IEEE Trans. PAMI , vol. 11, pp. 674-692, no. 7, July 1989.

## dwt

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### Purpose

Discrete wavelet transform

### Synopsis

$[y,ty] = \text{dwt}(H,tH,G,tG,x)$

$[y,ty] = \text{dwt}(H,tH,G,tG,x,rm)$

### Description

$[y,ty] = \text{dwt}(H,tH,G,tG,x,rm)$  returns the discrete wavelet decomposition coefficients  $y$  of the signal  $x$  up to the resolution level  $rm$  (optional). the details of the signal  $x$  are stored in  $y$  from coarse to fine resolution.  $H$  and  $G$  are the impulse responses of the analysis lowpass and highpass filters which start at time  $tH$  and  $tG$ , respectively, and  $ty$  is a matrix which contains information on  $y$  in the following way:

- 1st row: delays of the  $y$  signals
- 2nd row: length of the  $y$  signals

$rm$  is the highest resolution level which is by default equal to  $\text{LOWPOW2}(x)$ .

### Algorithm

The discrete wavelet decomposition function is computed using the pyramid algorithm which is described by Mallat in [1]. This algorithm consists in decomposing the signal spectrum into two equal (orthogonal case) or unequal (biorthogonal case) frequency bands using a lowpass ( $H$ ) and a highpass ( $G$ ) quadrature mirror filters (QMFs). The results of  $H$  and  $G$  are decimated by 2 and are called the approximations vector and the details vector, respectively. Then, the same procedure is applied again to the approximations vector only as many times -resolution levels- as it is possible. The wavelet coefficients localized outside the signal support are computed by zero padding the signal. As a result, the number of wavelet coefficients is greater than the signal length.

### See also

IDWT in the GDR Wavelet Toolbox.

### Authors

J-C. Pesquet and H. Krim

### References

- [1] S. G. Mallat, "A Theory for Multiresolution Signal Decomposition: The Wavelet Representation," IEEE Trans. PAMI, vol. 11, pp. 674-692, no. 7, July 1989.

## **ibdwt**

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### **Purpose**

Inverse discrete wavelet decomposition on the interval

### **Synopsis**

$x = \text{ibdwt}(h,g,Ml,iAl,Mr,iAr,y)$

$x = \text{ibdwt}(h,g,Ml,iAl,Mr,iAr,y,rm)$

### **Description**

$x = \text{ibdwt}(h,g,Ml,iAl,Mr,iAr,y,rm)$  computes the inverse discrete wavelet decomposition on the interval.  $h$  and  $g$  are the lowpass and highpass impulse responses of the synthesis filters,  $Ml$  and  $Mr$  are the matrices used at left and right boundaries to reconstruct the signal,  $iAl$ ,  $iAr$  are the inverses of the preconditionning matrices for left and right data boundaries,  $y$  is the vector of the wavelet coefficients,  $rm$  is the maximum resolution level of decomposition and  $x$  is the reconstructed signal.

### **See also**

BDWTD, IBWAVELD, IBWAVERD in the GDR Wavelet Toolbox.

### **Authors**

J-C. Pesquet and H. Krim

### **References**

[1] A. Cohen and I. Daubechies, "Wavelets on the interval and Fast Wavelet Transform", Applied and Computational Harmonic Analysis 1, 54-81, 1993.

## **ibwaveld**

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### **Purpose**

Generate the left boundary matrices used in the inverse wavelet decomposition on the interval

### **Synopsis**

$[Ml,iAl] = \text{ibwaveld}(h,g,hl,gl,Al)$

### **Description**

$[Ml,iAl] = \text{ibwaveld}(h,g,hl,gl,Al)$  computes the matrices allowing to reconstruct the signal at the left boundary in the inverse wavelet decomposition on the interval function.  $h$  and  $g$  are the impulse responses of the synthesis lowpass and highpass filters and  $hl$ ,  $gl$  and  $Al$  are the matrices obtained from BWAVELD.

### **See also**

IBDWTD, IBWAVERD in the GDR Wavelet Toolbox.

### **Authors**

J-C. Pesquet and H. Krim

### **References**

[1] A. Cohen and I. Daubechies, "Wavelets on the interval and Fast Wavelet Transform", Applied and Computational Harmonic Analysis 1, 54-81, 1993.

## **ibwaverd**

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### **Purpose**

Generate the right boundary matrices used in the inverse wavelet decomposition on the interval.

### **Synopsis**

$[Mr, iAr] = \text{ibwaverd}(h, g, hr, gr, Ar)$

### **Description**

$[Mr, iAr] = \text{ibwaverd}(h, g, hr, gr, Ar)$  computes the matrices allowing to reconstruct the signal at the right boundary in the inverse wavelet decomposition on the interval.  $h$  and  $g$  are the impulse responses of the synthesis lowpass and highpass filters.  $hr$ ,  $gr$  and  $Ar$  are the matrices obtained from `BWAVERD`.

### **See also**

IBDWTD, IBWAVELD in the GDR Wavelet Toolbox.

### **Authors**

J-C. Pesquet and H. Krim

### **References**

[1] A. Cohen and I. Daubechies, "Wavelets on the interval and Fast Wavelet Transform", *Applied and Computational Harmonic Analysis* 1, 54-81, 1993.

## idwt

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### Purpose

Inverse discrete wavelet decomposition

### Synopsis

$x = \text{idwt}(H,tH,G,tG,y,ty,Lx)$

### Description

$x = \text{idwt}(H,tH,G,tG,y,ty,Lx)$  computes the inverse discrete wavelet decomposition of a vector  $y$  where the details of the original signal  $x$ , whose length is  $Lx$ , are stored from coarse to fine resolution.  $H$  and  $G$  are the impulse responses of the synthesis lowpass and highpass filters starting at time  $tH$  and  $tG$ , respectively.  $ty$  is the matrix which contains information on  $y$  such that :

- 1st row: delays of the  $y$  signals
- 2nd row: length of the  $y$  signals

### Algorithm

The reconstruction of the signal from its wavelet decomposition coefficients is achieved by up-sampling and convolving the approximations and details vectors of the highest resolution level with the synthesis lowpass and highpass filters, respectively. These two results are then summed up to give a new approximations vector at a lower resolution level. This procedure is repeated as many times as the number of resolution levels which has been used to decompose the signal.

### See also

DWT in the GDR Wavelet Toolbox.

### Authors

J-C. Pesquet and H. Krim

### References

- [1] S. G. Mallat, "A Theory for Multiresolution Signal Decomposition: The Wavelet Representation," IEEE Trans. PAMI, vol. 11, pp. 674-692, no. 7, July 1989.

## ipdwt

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### Purpose

Inverse periodic discrete wavelet decomposition

### Synopsis

$x = \text{ipdwt}(H,G,y)$

$x = \text{ipdwt}(H,G,y,rm)$

### Description

$x = \text{ipdwt}(H,G,y)$  computes the inverse periodic discrete wavelet decomposition of the vector  $y$  where the wavelet coefficients are stored in the increasing resolution order.  $H$  and  $G$  are the frequency responses of the lowpass and highpass synthesis filters, respectively.  $H$  and  $G$  may be chosen different at each resolution level.

$x = \text{ipdwt}(H,G,y,rm)$  computes the inverse periodic discrete wavelet decomposition of the vector  $y$  up to the resolution  $rm$ .  $rm$  is the highest resolution which is by default equal to  $\text{LOWPOW2}(X)$ .

### Algorithm

See PDWT algorithm.

### See also

PDWT in the GDR Wavelet Toolbox.

### Authors

J-C. Pesquet and H. Krim

### References

- [1] S. G. Mallat, "A Theory for Multiresolution Signal Decomposition: The Wavelet Representation," IEEE Trans. PAMI, vol. 11, pp. 674-692, no. 7, July 1989.

## ipdwt\_it

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### Purpose

Inverse periodic discrete wavelet decomposition computed over one iteration

### Synopsis

$x = \text{ipdwt\_it}(H,G,Hx,Gx,r,Lx)$

### Description

$x = \text{ipdwt\_it}(H,G,Hx,Gx,r,Lx)$  computes one iteration of the inverse periodic discrete wavelet decomposition of the vector  $y$ .  $H$  and  $G$  are the frequency responses of the synthesis lowpass and highpass filters used at each resolution level,  $Hx$  and  $Gx$  are the approximations and details, respectively,  $r$  is the resolution level and  $Lx$  is the length of the original signal.  $x$  is the reconstructed signal at the resolution  $r-1$  (approximation or detail).

### Algorithm

See IPDWT algorithm.

### See also

IPDWT in the GDR Wavelet Toolbox.

### Authors

J-C. Pesquet and H. Krim

### References

[1] S. G. Mallat, "A Theory for Multiresolution Signal Decomposition: The Wavelet Representation," IEEE Trans. PAMI, vol. 11, pp. 674-692, no. 7, July 1989.

## pcdwt

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### Purpose

Redundant periodic discrete wavelet decomposition

### Synopsis

`pcdwt(H,tH,G,tG,x)`

`pcdwt(H,tH,G,tG,x,rm)`

### Description

`y = pcdwt(H,tH,G,tG,x)` computes the redundant periodic discrete wavelet decomposition of the signal `x` up to the resolution level `rm` (default value: `LOWPOW(x)`). `y` is a vector where all the details of signal `x` are stored from coarse to fine resolution. `H` and `G` are the impulse responses of the lowpass and highpass analysis filters starting at time `tH` and `tG`, respectively.

`y = pcdwt(H,tH,G,tG,x,rm)` computes the redundant periodic discrete wavelet decomposition of signal `x` up to the resolution level `rm`.

### Algorithm

The discrete wavelet decomposition function is computed using the pyramid algorithm which is described by Mallat in [1]. This algorithm consists in decomposing the signal spectrum into two equal frequency bands using a lowpass (`H`) and a highpass (`G`) quadrature mirror filters (QMFs). They are called the approximations vector and the details vector, respectively. Then, the same procedure is applied again to the approximations vector as many times -resolution levels- as it is possible. But, the results of `H` and `G` are not decimated by 2, and therefore the number of generated wavelet coefficients increases exponentially with the number of resolution levels.

### See also

CDWT, PDWT in the GDR Wavelet Toolbox.

### Authors

J-C. Pesquet and H. Krim

### References

[1] S. G. Mallat, "A Theory for Multiresolution Signal Decomposition: The Wavelet Representation," IEEE Trans. PAMI, vol. 11, pp. 674-692, no. 7, July 1989.

## pdwt

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### Purpose

Periodic discrete wavelet decomposition

### Synopsis

$y = \text{pdwt}(H,G,x)$

$y = \text{pdwt}(H,G,x,rm)$

### Description

$y = \text{pdwt}(H,G,x)$  computes the discrete wavelet decomposition of the signal  $x$ . The wavelet coefficients are stored in  $y$  in the increasing resolution order.  $H$  and  $G$  are the frequency responses of the analysis lowpass and highpass filters. The highest resolution level is set by default to  $\text{LOWPOW2}(x)$ .

$y = \text{pdwt}(H,G,x,rm)$  computes the discrete wavelet decomposition,  $y$ , of the signal  $x$  up to the resolution  $rm$ .

### Algorithm

The discrete wavelet decomposition function is computed using the pyramid algorithm which is described by Mallat in [1]. This algorithm consists in decomposing the signal spectrum into two equal (orthogonal case) or unequal (biorthogonal case) frequency bands using a lowpass ( $H$ ) and a highpass ( $G$ ) quadrature mirror filters (QMFs). The results of  $H$  and  $G$  are decimated by 2 and are called the approximations vector and the details vector, respectively. Then, the same procedure is applied again to the approximations vector only, as many times -resolution levels- as it is possible. In this decomposition, the linear convolutions are replaced by circular convolutions, which is equivalent to periodizing the analyzed signal. This procedure allows to reduce the boundary effects.

### See also

IPDWT in the GDR Wavelet Toolbox.

### Authors

J-C. Pesquet and H. Krim

### References

[1] S. G. Mallat, "A Theory for Multiresolution Signal Decomposition: The Wavelet Representation," IEEE Trans. PAMI, vol. 11, pp. 674-692, no. 7, July 1989.

## pdwt\_it

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### Purpose

Periodic discrete wavelet decomposition computed over one iteration

### Synopsis

$[HX, GX] = \text{pdwt\_it}(H,G,X,r,Lx)$

### Description

$[HX, GX] = \text{pdwt\_it}(H,G,X,r,Lx)$  computes one iteration of the periodic discrete wavelet decomposition of signal  $x$ , expressed in the spectral domain by  $X$  (Fourier Transform of  $x$ ).  $H$  and  $G$  are the frequency responses of the lowpass and highpass analysis filters, used at each level of the analysis and stored in decreasing resolution order.  $r$  is the desired resolution level,  $Lx$  the length of the original signal.  $HX$  and  $GX$  contain the approximation and details coefficients of  $X$  in the spectral domain. In this decomposition, the convolutions are replaced by circular convolutions which is equivalent to periodizing the analyzed signal. This procedure allows to reduce the boundary effects.

### Algorithm

See PDWT algorithm.

### See also

PDWT in the GDR Wavelet Toolbox.

### Authors

J-C. Pesquet and H. Krim

### References

[1] S. G. Mallat, "A Theory for Multiresolution Signal Decomposition: The Wavelet Representation," IEEE Trans. PAMI, vol. 11, pp. 674-692, no. 7, July 1989.

## **Wavelet Packet Decompositions**

## Contents

<b>dwpt :</b>	Discrete wavelet packet decomposition
<b>idwpt :</b>	Inverse discrete wavelet packet decomposition
<b>pdwpt :</b>	Periodic discrete wavelet packet decomposition
<b>ipdwpt :</b>	Inverse periodic discrete wavelet packet decomposition

## dwpt

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### Purpose

Discrete wavelet packet decomposition

### Synopsis

$[V_x, \text{INFO}, tV_x] = \text{dwpt}(h, th, g, tg, x, \text{criterion})$

$[V_x, \text{INFO}, tV_x] = \text{dwpt}(h, th, g, tg, x, \text{criterion}, rm)$

$[V_x, \text{INFO}, tV_x] = \text{dwpt}(h, th, g, tg, x, \text{criterion}, rm, \text{enerthresh})$

$[V_x, \text{INFO}, tV_x] = \text{dwpt}(h, th, g, tg, x, \text{criterion}, rm, \text{enerthresh}, \text{entthresh})$

### Description

$[V_x, \text{INFO}, tV_x] = \text{dwpt}(h, th, g, tg, x, \text{criterion})$  computes the discrete wavelet packet decomposition vector  $V_x$  of signal  $x$  according to the cost function "criterion".  $h$  and  $g$  are the impulse responses of the analysis lowpass and highpass filters starting at time  $th$  and  $tg$ , respectively.  $\text{INFO}$  is a matrix which serves to manage the discrete wavelet packet decomposition. Each column contains information on each signal of  $V_x$ . There are  $2^{(rm+1)-1}$  columns. Each row contains information of different types :

- 1st row: index of the beginning of the signal in  $V_x$
- 2nd row: signal length

$tV_x$  is a vector containing the delays of  $V_x$  signals

$[V_x, \text{INFO}, tV_x] = \text{dwpt}(h, th, g, tg, x, \text{criterion}, rm)$  computes the discrete wavelet packet decomposition of signal  $x$  up to the resolution  $rm$ .

$[V_x, \text{INFO}, tV_x] = \text{dwpt}(h, th, g, tg, x, \text{criterere}, rm, \text{enerthresh})$  stops the discrete wavelet packet decomposition if the energy of the wavelet packet coefficients is below the energy threshold "enerthresh".

$[V_x, \text{INFO}, tV_x] = \text{dwpt}(h, th, g, tg, x, \text{criterion}, rm, \text{enerthresh}, \text{entthresh})$  stops the discrete wavelet packet decomposition if the energy of the wavelet packet coefficients is below the entropic threshold "entthresh".

### Algorithm

The wavelet packet bases form a collection -library- of orthonormal bases composed of functions of the form  $W_n(2^l x - k)$  where  $l, k \in \mathbb{Z}$  and  $n \in \mathbb{N}$ . Each element of the library is determined by the following subset of indices :

- 1) a scaling parameter  $l$
- 2) a localization parameter  $k$
- 3) an oscillating parameter  $n$

This algorithm, which is well explained in [1] and [2], consists in decomposing the signal on the best wavelet packet basis according to a certain criterion or cost function.

### See also

IDWPT in the GDR Wavelet Toolbox.

## **Authors**

H. Carfantan, H. Krim and J-C. Pesquet.

## **References**

- [1] R. R. Coifman and M. V. Wickerhauser, Best-adapted wavelet packet bases, preprint, Yale University.
- [2] Yves Meyer and M. V. Wickhauser, Acoustic signal compression with wavelet packets, preprint, Yale University, August, 1989.

## idwpt

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### Purpose

Inverse discrete wavelet packet decomposition

### Synopsis

$z = \text{idwpt}(H, G, tH, tG, Vx, \text{INFO}, tVx, Lx)$

### Description

$z = \text{idwpt}(H, G, tH, tG, Vx, \text{INFO}, tVx, Lx)$  reconstructs the signal  $z$  of length  $Lx$  from its wavelet packet decomposition coefficients  $Vx$ .  $H$  and  $G$  are the impulse responses of the synthesis lowpass and highpass filters starting at time  $tH$  and  $tG$ , respectively.  $tVx$  is a vector containing the delays of  $Vx$  signals and  $\text{INFO}$  is a matrix containing information on  $Vx$  in the following way:

- 1st row: index of the beginning of  $Vx$
- 2nd row: length of  $Vx$  signals

### Algorithm

The reconstruction of the signal from its wavelet packet decomposition coefficients is achieved by up-sampling and convolving the approximations and details vectors with the synthesis lowpass and highpass filters, respectively. These two results are summed up to give an approximations vector of lower resolution. Then the same procedure is repeated as many times as the number of resolutions levels and following -from right to left - the same tree which has been used in the decomposition.

### See also

DWPT in the GDR Wavelet Toolbox.

### Authors

H. Carfantan, H. Krim and J-C. Pesquet.

### References

- [1] R. R. Coifman and M. V. Wickerhauser, Best-adapted wavelet packet bases, preprint, Yale University.
- [2] Yves Meyer and M. V. Wickhauser, Acoustic signal compression with wavelet packets, preprint, Yale University, August, 1989.

## ipdwpt

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### Purpose

Inverse periodic discrete wavelet packet decomposition

### Synopsis

$x = \text{ipdwpt}(H,G,V_x,INFO,rm)$

### Description

$x = \text{ipdwpt}(H,G,V_x,INFO)$  computes the periodic discrete wavelet packet decomposition of a vector  $V_x$  where the wavelet packet coefficients are stored.  $H$  and  $G$  are frequency responses of the lowpass and highpass synthesis filters.  $INFO$  is a matrix containing information on the decomposition:

- 1st row: index of the beginning of the signals in  $V_x$
- 2nd row: signals length

$x = \text{ipdwpt}(H,G,V_x,INFO,rm)$  computes the periodic discrete wavelet packet decomposition up to the resolution level  $rm$ .

### Algorithm

See DWPT algorithm

### See also

PDWPT in the GDR Wavelet Toolbox.

### Authors

H. Carfantan, H. Krim and J-C. Pesquet.

### References

- [1] R. R. Coifman and M. V. Wickerhauser, Best-adapted wavelet packet bases, preprint, Yale University.
- [2] Yves Meyer and M. V. Wickhauser, Acoustic signal compression with wavelet packets, preprint, Yale University, August, 1989.

## pdwpt

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### Purpose

Periodic discrete wavelet packet decomposition

### Synopsis

$[Vx,INFO] = \text{pdwpt}(H,G,x,\text{ncost})$

$[Vx,INFO] = \text{pdwpt}(H,G,x,\text{ncost},\text{rm})$

$[Vx,INFO] = \text{pdwpt}(H,G,x,\text{ncost},\text{rm},\text{enerthresh})$

$[Vx,INFO] = \text{pdwpt}(H,G,x,\text{ncost},\text{rm},\text{enerthresh},\text{entthresh})$

### Description

$[Vx,INFO] = \text{pdwpt}(H,G,x,\text{ncost})$  computes the periodic discrete wavelet packet decomposition of signal  $x$  relative to the cost function "ncost".  $H$  and  $G$  are the frequency responses of the analysis lowpass and highpass filters.  $INFO$  is a matrix which manages the wavelet packet decomposition: each column contains information on each signal of  $Vx$ . There are  $2^{(rm+1)}-1$  columns. Each row contains information of a different type:

- 1st row : index of the beginning of the signal in  $Vx$ .
- 2nd row : signal length.

$[Vx,INFO] = \text{pdwpt}(H,G,x,\text{ncost},\text{rm})$  computes the periodic discrete wavelet packet decomposition up to the resolution  $rm$ .  $rm$  is the highest resolution (default value:  $\text{LOWPOW2}(x)$ ).

$[Vx,INFO] = \text{pdwpt}(H,G,x,\text{ncost},\text{rm},\text{enerthresh})$  computes the periodic discrete wavelet packet decomposition up to the resolution  $rm$ . If the energy of the signal is below the energy threshold "enerthresh" then the signal is no more decomposed.

$[Vx,INFO] = \text{pdwpt}(H,G,x,\text{ncost},\text{rm},\text{enerthresh},\text{entthresh})$  computes the periodic discrete wavelet packet decomposition up to the resolution  $rm$ . if the energy of the signal is below the energy threshold then the signal is no more decomposed. The same thing happens if the entropic cost of the signal is below the entropic threshold "entthresh".

### Algorithm

See DWPT algorithm

### See also

IPDWPT in the GDR Wavelet Toolbox.

### Authors

H. Carfantan, H. Krim and J-C. Pesquet.

## References

- [1] R. R. Coifman and M. V. Wickerhauser, Best-adapted wavelet packet bases, preprint, Yale University.
- [2] Yves Meyer and M. V. Wickhauser, Acoustic signal compression with wavelet packets, preprint, Yale University, August, 1989.

## **Design Tools**

## Contents

<b>exmeyer</b> :	Example function to construct Meyer's wavelet
<b>freqcqf</b> :	Complex conjugate frequency response of a filter
<b>frmeyer</b> :	Frequency response of Meyer's QMF filter
<b>frsplis</b> :	Frequency responses of lowpass B-spline symmetric wavelet filters
<b>gene_pw</b> :	Generation of filters H and G for periodic discrete wavelet decomposition
<b>irdaub</b> :	Impulse response of Daubechies wavelet filters
<b>irsplis</b> :	Impulse response of lowpass B-spline symmetric wavelet filters
<b>pdwav</b> :	Generation of discrete periodic wavelets and scaling functions
<b>pdwavp</b> :	Generation of discrete periodic wavelet packets
<b>qmf</b> :	Conjugate quadrature mirror filter

## **exmeyer**

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### **Purpose**

Example function to construct Meyer's wavelet

### **Synopsis**

$y = \text{exmeyer}(f)$

### **Description**

$y = \text{exmeyer}(f)$  constructs Meyer's wavelet .  $f$  is the vector of inputs between 0 and 1.

### **Algorithm**

Meyer's wavelet is reconstructed using the following formula :

$$y = 1 - \exp(al/(f-1)^2), \quad al < 0, \quad 0 \leq f \leq 1.$$

If  $f = 0$  then  $y = 1/\sqrt{2}$  and if  $f = 1$  then  $y = 1$ .

### **See also**

FRMEYER in the GDR Wavelet Toolbox.

### **Authors**

J-C. Pesquet and H. Krim

## freqcqf

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### Purpose

Complex conjugate frequency response of a filter

### Synopsis

$G = \text{freqcqf}(H,P,t)$

### Description

$G = \text{freqcqf}(H,P,t)$  computes the frequency response  $G$  which is the complex conjugate of filter  $H$ ,  $P$  is the delay (default value: 0) and  $t$  is the phase shift (default value: 0).

### Algorithm

The frequency response  $G$  is computed using the following formula :

$$G(F) = \exp(-j(2P+1)F+t) \text{ conj}(H(F+1/2))$$

where :        H: lowpass filter samples  
                  F: frequency  
                  P: delay  
                  t: phase shift.

### See also

QMF in the GDR Wavelet Toolbox.

### Authors

J-C. Pesquet and H. Krim

## frmeyer

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### Purpose

Frequency response of Meyer's QMF filter

### Synopsis

$H = \text{frmeyer}(N,ga,ep)$

### Description

$H = \text{frmeyer}(N,ga,ep)$  returns the frequency response of the lowpass QMF filter corresponding to Meyer's wavelet.  $N$  is the number of samples,  $ga$  is function allowing to construct the wavelet, it must return values between  $1/\sqrt{2}$  and 1 for arguments between 0 and 1 and  $ep$  is a parameter between 0 and  $1/6$  : a small value ensures a good frequency localization (default value :  $1/6$ ).

### See also

EXMEYER in the GDR Wavelet Toolbox.

### Authors

J-C. Pesquet and H. Krim

## **frsplis**

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### **Purpose**

Frequency responses of lowpass B-spline symmetric wavelet filters

### **Synopsis**

$H = \text{frsplis}(N,r)$

### **Description**

$H = \text{frsplis}(N,r)$  computes the frequency response of lowpass QMF filters corresponding to B-spline symmetric wavelets with order  $r$ .  $N$  is the number of samples of  $H$ .

### **See also**

IRSPLIS in the GDR Wavelet Toolbox.

### **Authors**

J-C. Pesquet and H. Krim

## gene\_pw

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### Purpose

Generation of filters H and G for periodic discrete wavelet decomposition

### Synopsis

$[H,G] = \text{gene\_pw}(N,TYP)$

$[H,G] = \text{gene\_pw}(N,TYP,RM)$

### Description

$[H,G] = \text{gene\_pw}(N,TYP)$  returns the frequency responses of the lowpass and highpass filters which allow to reconstruct each multiresolution level. N is the data length and TYP is the wavelet type. The resolution level is by default set to NEXTPOW2(N).

$[H,G] = \text{gene\_pw}(N,TYP,RM)$  allows to specify the resolution level RM.

### See also

QMF in the GDR Wavelet Toolbox.

### Authors

J-C. Pesquet and H. Krim

## **irdaub**

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### **Purpose**

Impulse response of Daubechies wavelet filters

### **Synopsis**

$h = \text{irdaub}(n)$

### **Description**

$h = \text{irdaub}(n)$  computes the impulse responses of the lowpass QMF filters with minimum phase and length  $n$ , allowing to generate Daubechies wavelets.  $n$  should be an integer number greater than 0.  $n = 1$  corresponds to the Haar basis.  $h$  is  $2*n$  long.

### **See also**

IRSPLIS in the GDR Wavelet Toolbox.

### **Authors**

J-C. Pesquet and H. Krim

### **References**

[1] I. Daubechies, "Orthonormal Bases of Compactly Supported Wavelets," Comm. in Pure and Applied Math., vol. 41, pp. 909-996, 1988.

## **irsplis**

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### **Purpose**

Impulse response of lowpass B-spline symmetric wavelet filters

### **Synopsis**

$[h,th] = \text{irsplis}(N,r)$

### **Description**

$[h,th] = \text{irsplis}(N,r)$  computes the impulse responses of lowpass QMF filters allowing to generate B-spline symmetric wavelets.  $N$  is the number of samples (should be chosen large enough to avoid truncation troubles),  $r$  is the spline order and  $th$  is the starting time of  $h$ .

### **See also**

IRDAUB, FRSPILIS in the GDR Wavelet Toolbox.

### **Authors**

J-C. Pesquet and H. Krim

## pdwav

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### Purpose

Generation of discrete periodic wavelets and scaling functions

### Synopsis

$x = \text{pdwav}(H,G,N,R,T,TYP)$

### Description

$x = \text{pdwav}(H,G,N,R,T)$  generates the discrete periodic wavelets and scaling functions corresponding to the lowpass and highpass filters whose frequency responses are given by  $H$  and  $G$ .  $N$  is the length of  $X$ ,  $R$  is the highest resolution and  $T$  is the time position for the frequency bandwidth from 0 to  $N/2^{(R-1)}$ . By choosing  $R$  large enough the continuous functions are approximated.

$x = \text{pdwav}(H,G,N,R,T,TYP)$  allows to specify the wavelet type.  $TYP$  is the wavelet type. if  $TYP = 'a'$ , then the scaling function is computed, otherwise the wavelet function is computed.

### See also

GENE\_PW in the GDR Wavelet Toolbox.

### Authors

J-C. Pesquet and H. Krim

### References

[2] I. Daubechies, "Orthonormal Bases of Compactly Supported Wavelets," Comm. in Pure and Applied Math., vol. 41, pp. 909-996, 1988.

## pdwavp

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### Purpose

Generation of discrete periodic wavelet packets

### Synopsis

$x = \text{pdwavp}(H,G,N,R,p,t)$

### Description

$x = \text{pdwavp}(H,G,N,R,p,t)$  generates the discrete wavelet packets corresponding to the lowpass and highpass filters whose frequency responses are given by  $H$  and  $G$ .  $N$  is the signal length,  $R$  is the maximum resolution level,  $p$  is the position of the frequency band from 0 to  $2^R - 1$  and  $t$  is the time position of the frequency band from 0 to  $N/2^R - 1$ .

### See also

PDWAV in the GDR Wavelet Toolbox.

### Authors

H. Carfantan, H. Krim and J-C. Pesquet.

### References

- [1] R. R. Coifman and M. V. Wickerhauser, Best-adapted wavelet packet bases, preprint, Yale University, February.
- [2] Yves Meyer and M. V. Wickhauser, Acoustic signal compression with wavelet packets, preprint, Yale University, August, 1989.

## **qmf**

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### **Purpose**

Conjugate quadrature mirror filter

### **Synopsis**

$g = \text{qmf}(h)$

### **Description**

$g = \text{qmf}(h)$  computes the conjugate quadrature mirror filter of a filter whose impulse response is given by  $h$ .

### **See also**

FREQCQF in the GDR Wavelet Toolbox.

### **Authors**

J-C. Pesquet and H. Krim